Partial Information Target Defense Game

Daigo Shishika[1], Dipankar Maity[2], and Michael Dorothy[3]

***Abstract*— We formulate a scenario in which an autonomous defender is tasked with intercepting an intruder that tries to reach a circular target region. This is a variant of the target defense problem proposed by Isaacs as a pursuit-evasion game. Unlike the original target guarding problem and its various extensions, we consider the effect of partial information by imposing sensing limitation to the robots. We analyze the game by decomposing it into three game phases: deployment, asymmetric information, and engagement phase. Focusing on a particular parameter regime, we propose a simple defender strategy together with the lower bound on the probability that it wins the game. The defender strategy in each phase is constructed so that the subsequent phase starts in a desired initial configuration. The proposed problem is rich in terms of the parameter regimes that it contains, and thus is expected to be a useful platform in exploring effective control policies.**

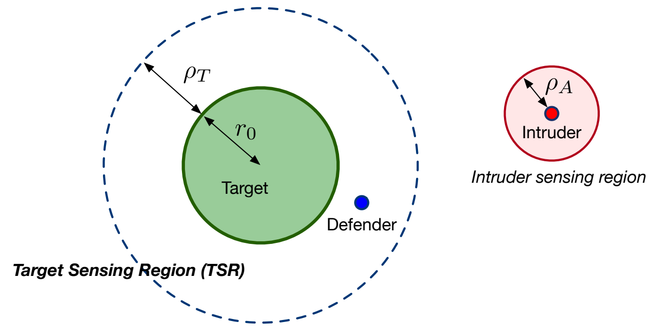

Fig. 1: Illustration of the partial-information target-defense game.

I. Introduction

The use of robotic agents for security applications including patrolling, monitoring, or securing of regions, is becoming more and more realistic with the advancements in automation and robotics. When we study adversarial scenarios involving robots or agents with conflicting goals, the scenarios can be modeled as non-cooperative games. More specifically, scenarios with evasive targets which need to be detected, intercepted, or surrounded by other robots are often formulated as pursuit-evasion games (PEGs).

Since the seminal work by Isaacs [1], many different variants of PEGs have been proposed to model and study security-related scenarios. A class of PEGs relevant to this paper is called the *target guarding problem* [1], in which a defender / pursuer is tasked with intercepting an intruder / evader before it reaches a target region. Many versions of this problem is studied under the name *target-attacker-defender* game [2]–[4], or *reach-avoid game* [5]–[17], and with connections to defense and security application including coast-line guarding or boarder defense [18]–[21].

A common assumption in the aforementioned works is the full-state information, i.e., all the agents in the game has access to the states of all other agents. This is a critical assumption to enable us leverage differential-game techniques to derive equilibrium strategies [22]. However, full-state information is unreasonable in many realistic security scenarios.

This paper proposes a variant of the target guarding problem that incorporates the aspect of partial information. Specifically, we assume limited sensor footprint for the agents, and therefore the agents have to consider not only the positional advantage but also the informational advantage when deciding on their movement. Similar sensing constraints have been studied in [23], [24], however, the problem is more complex with the addition of the target.

We approach the problem by decomposing the game into different phases. The key idea in deriving the strategies is that, in each phase, the players will select their strategies so that the next phase will start in a favorable initial configuration.

The contributions of the paper are: (i) the formulation of the partial-information target-defense game (PITDG), which incorporates the sensing aspect into the target guarding problem; (ii) the solution approach where the problem is decomposed into different game phases with direct connections between adjacent phases; and (iii) a defender strategy that gives an upper bound on the intruder's success rate, which will be useful as a benchmark in the future study.

II. Preliminaries

A. Problem Formulation

We consider a target guarding problem in $\mathbb{R}^2$ where an intruder tries to breach the boundary of a circular target region $\mathcal{R}_T = \{x \in \mathbb{R}^2 | \; \|x\| \leq r_0\}$ without being captured by a defender. The defender must capture the intruder before it penetrates the target boundary $\partial\mathcal{R}_T$. Let $x_A(t)$ and $x_D(t)$ denote the positions of the intruder and the defender at time t. Both the defender and the intruder have first-order dynamics. Without the loss of generality, we assume the defender and

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the intruder to have the speed limit of 1 and ν, respectively, with the constraint $\nu \leq 1$.

The intruder is able to sense the defender only if it is within a distance ρ_A. Let $\mathcal{B}_{\rho_i}(x_i) = \{x \in \mathbb{R}^2 | \|x - x_i\| \leq \rho_i\}$ denote a disk with radius ρ_i centered at x_i. Then $\mathcal{B}_{\rho_A}(x_A(t))$ denotes the sensing region of the intruder at time t. The target is also equipped with a limited-range sensor, and its sensing region is the annulus with width ρ_T, i.e., $\mathcal{B}_{r_0+\rho_T}(0) \setminus \mathcal{B}_{r_0}(0)$, which we call the ***target-sensing region (TSR)***.

The target and the defender jointly act as a team, and hence, *whenever the target senses the presence of the intruder, it immediately relays that sensed information to the defender* so that the defender can take necessary actions. Importantly, the intruder is aware of the location of the target and its sensing radius, so it knows when it has been sensed.

The terminal condition of this game is when either the defender *captures* the intruder $\|x_A(t) - x_D(t)\| = 0$, or the intruder *breaches* the boundary of the target $\|x_A(t)\| < r_0$, whichever that occurs first.

The strategies of the players are to choose their speeds and heading angles based on the information available to them. We consider the problem of whether the intruder can breach the target or the defender can successfully capture the intruder as a *Game of Kind*. We will see that the information limitation results in the strategies being probabilistic.

Problem. *What are the intruder/defender strategies that maximize their respective probabilities of win?*

The behaviors of the agents and the overall outcome of this partial information game heavily depends on the game parameters ρ_A, ρ_T and ν. Since the treatment of the entire parameter space is beyond the scope and page limitation of this paper, here we focus on a specific parameter regime given by the following assumptions:

(A1) $\rho_T < r_0 \nu$, and
(A2) $\rho_A < \rho_T$.

The first assumption avoids the degenerate game where the TSR is so large that the defender can always win. The second assumption helps us restrict the types of possible intruder strategies. The implications of these assumptions will be discussed further as we move forward.

B. Game Phases

In the following we define three game phases which together completely characterize the entire game.

Definition 1 (Deployment Phase)**.** *Every scenario starts in the* ***deployment phase****, in which the intruder is outside of the TSR. In this phase neither agent has its opponent in its sensing range, and therefore the agents do not have the information about the opponent's location.*

Denote the beginning of the deployment phase (and the game itself) as $t = 0$. The deployment phase ends when either the intruder enters the TSR, or the defender enters the sensing region of the intruder.

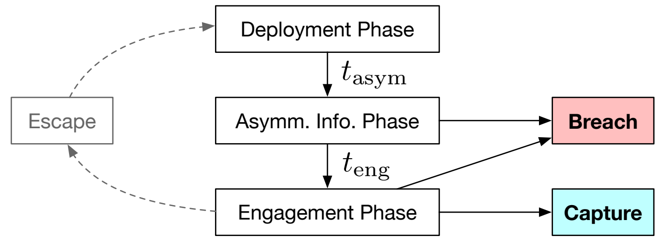

Fig. 2: Game phases and possible transitions considered in this paper.

There are many possible initial conditions which could result in interesting games. For example, if the defender is sensed by the intruder far from the target, even an intruder with lower speed than the defender can intelligently exploit his knowledge that the target is poorly guarded.

However, we will restrict ourselves to a class of initial conditions where $x_D(0) = 0$, $\|x_A(0)\| > M$ with sufficiently large M, and the class of defender strategies such that it will not be sensed by the intruder that is outside of the TSR. Specifically, it is sufficient if the defender's position in the deployment phase satisfies

$$\|x_D\| \leq r_0 + \rho_T - \rho_A. \tag{1}$$

Under this class of defender strategy, the deployment phase terminates only when the intruder enters the TSR.

Definition 2 (Asymmetric Information Phase)**.** *The* ***asymmetric information phase*** *is the duration of the game in which only one agent has the information about the other.*

In general, there are two distinct ways this phase can occur. First, when the intruder reaches the TSR before sensing the defender, i.e.,

$$\|x_A(t)\| \leq r_0 + \rho_T, \tag{2a}$$

$$\|x_A(t) - x_D(t)\| > \rho_A. \tag{2b}$$

Since the defender and the target work as a team, the defender now has access to $x_A(t)$.

The second case is when the defender comes within the sensing region of the intruder but the intruder is outside the TSR. However, based on the restriction (1), this second case will not occur under the scope of this paper.

We will denote the beginning of the first asymmetric information phase as $t_{\text{asym}} > 0$. The asymmetric information phase continues so long as there is an asymmetry in the information. That is, one agent knows the location of the other, but not vice-versa.

Definition 3 (Engagement Phase)**.** *The* ***engagement phase*** *is the duration of the game in which both agents have access to their opponents' positions.*

That is, (2a) and $\|x_A(t) - x_D(t)\| \leq \rho_A$ both hold. The information structure in this phase is identical to that of a full-state information game. We will denote the the beginning of the first engagement phase as $t_{\text{eng}} > t_{\text{asym}}$. The engagement phase terminates when one of the following occurs:

1) Capture: $\|x_A(t) - x_D(t)\| = 0$ and $\|x_A(t)\| > r_0$,
2) Breach: $\|x_A(t)\| < r_0$,
3) Escape: $\|x_A(t)\| > r_0 + \rho_T$, or
4) Revert to asymmetric information phase.[1]

The game phases as well as the terminal states (breach, capture, and escape) are summarised in Fig. 2.

In the following sections we visit the game phases in a reverse chronological order to derive the strategies.

III. Engagement Phase

A. Apollonius Circle

The optimal strategies as well as the associated outcome in the full state information scenario can be obtained using a well-known geometry called the *Apollonius circle* [1]. For our problem, it is defined by the two points, x_A, x_D, and the speed ratio ν. It is a collection of points with constant distant ratio $\nu : 1$ from x_A and x_D. The center, x_C and the radius, r_C of the Apollonius circle is given as follows:

$$x_C = \alpha x_A - \beta x_D, \quad \text{and} \quad r_C = \gamma \|x_A - x_D\|, \tag{3}$$

where

$$\alpha = \frac{1}{1-\nu^2}, \quad \beta = \frac{\nu^2}{1-\nu^2}, \quad \text{and} \quad \gamma = \frac{\nu}{1-\nu^2}. \tag{4}$$

With $\nu < 1$, the point x_A is always contained in the Apollonius circle. In the limit $\nu \to 1$, the radius r_C approaches infinity, and the circle approaches the perpendicular bisector of x_A and x_D.

If the intruder and the defender both move towards a specific point on the Apollonius circle, then they will reach that point simultaneously. However, the intruder has a guarantee to reach any point inside the circle before the defender can. Therefore, the interior of the Apollonius circle describes a *dominance region* for the intruder. This property immediately leads to several sufficient conditions related to the full-state information engagement phase.

B. Sufficient Conditions

Lemma 1 (Breach)**.** *If the Apollonius circle intersects with the interior of the target, i.e.,*

$$\|x_C\| - r_C < r_0, \tag{5}$$

then, regardless of the strategy of the defender, there always exists an intruder strategy to breach the target.

Lemma 2 (Escape)**.** *If there exists a point on the Apollonius circle outside of the target region and TSR, i.e.,*

$$\|x_C\| + r_C > r_0 + \rho_T, \tag{6}$$

then, regardless of the strategy of the defender, there always exists an intruder strategy to evade from the defender.

The proofs of Lemmas 1-2 follow trivially from the properties of the Apollonius circle.

[1] In general, it is possible for the agents to take strategies which result in reversion from the engagement phase directly back to the asymmetric information phase. However, this does not occur for the strategies presented in this paper.

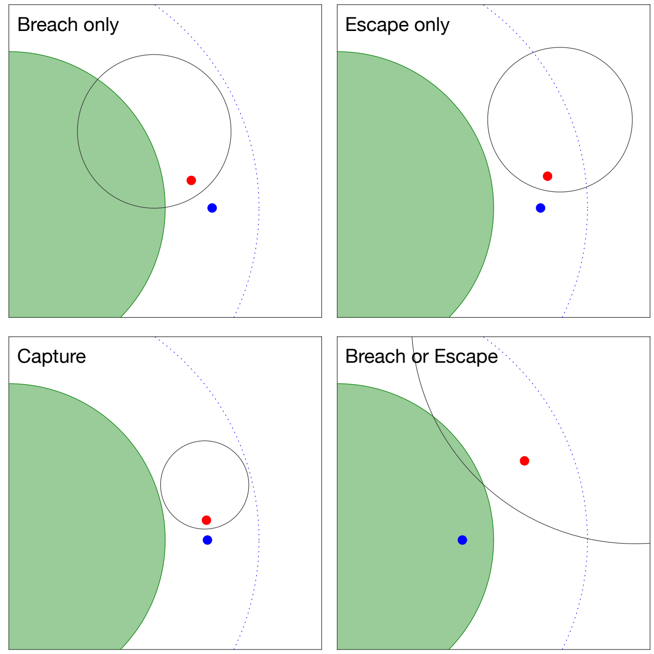

Fig. 3: The blue and red dots represent the positions of the defender and the intruder, respectively. The green region is a part of the circular target and the dotted blue line represents the target sensing boundary. The black (arc) circles represent (a part of) the Apollonius circle.

Corollary 1. *If the game parameters are such that*

$$\rho_T < 2\gamma\rho_A = \frac{2\nu\rho_A}{1-\nu^2} \tag{7}$$

then at $t = t_{eng}$ there always exists either an intruder strategy to breach or a strategy to evade.

Proof. At the moment the intruder senses the defender,

$$\|x_A(t_{\text{eng}}) - x_D(t_{\text{eng}})\| = \rho_A, \tag{8}$$

and thus, $r_C = \gamma\rho_A$. If the diameter of the Apollonius circle, $2\gamma\rho_A$, is larger than ρ_T, then the Apollonius circle cannot be contained inside the TSR, and either Lemma 1 or 2 applies. □

Lemma 3 (Guard)**.** *If the Apollonius circle does not intersect with the interior of the target, i.e.,*

$$\|x_C\| - r_C \geq r_0, \tag{9}$$

then, regardless of the strategy of the intruder, there always exists a defender strategy to prevent the intruder from breaching the target.

This follows trivially from [1] and the definition of the Apollonius circle. The counterpart of Corollary 1 is more subtle. While the intruder can focus solely on either breach or escape, the defender must simultaneously prevent both while closing to capture. In this paper, we present this as a conjecture and leave a formal proof for the extended journal version:

Conjecture 1 (Capture)**.** *If the Apollonius circle is contained in the interior of the TSR, then a defender strategy exists to capture the intruder.*

Conversely to Corollary 1, if

$$\rho_T > 2\gamma\rho_A = \frac{2\nu\rho_A}{1-\nu^2}, \tag{10}$$

then it is at least possible for the Apollonius circle to fit inside TSR, and Conjecture 1 may apply.

In subsequent sections, we will see that if (7) is satisfied, the intruder will always win (Corollary 3), while if (10) is satisfied, the game will end in one shot with a fixed probability of win for each agent (Corollary 2).

This section showed how the outcome of the game is determined by the initial condition of the engagement phase. In the following section, we consider strategies in the asymmetric information phase to achieve favorable initial conditions in the deployment phase.

IV. Asymmetric Information Phase

A. Directionality of Asymmetry

It is critical to know whether the asymmetric information phase begins with the defensive team sensing the intruder or the intruder sensing the defender . In this paper, we restrict ourselves to a class of defender strategy that satisfies (1) in the *deployment phase*, which ensures that prior to the intruder entering TSR, neither the intruder nor the defender has information about its opponent's position. Let R_{dep} denote the *deployment radius*, which is the radial position of the defender at t_{asym}. We will design R_{dep} in Sec. V, but in this section, it can be treated as part of the initial conditions for the asymmetric information phase. Due to (1), we have the following constraint:

$$R_{\text{dep}} < r_0 + \rho_T - \rho_A. \tag{11}$$

Under this constraint, the asymmetric information phase starts when the intruder reaches TSR. Moreover, in the asymmetric information phase, the defender knows the position of the intruder, whereas the intruder has not yet sensed the defender. Note that assumption (A2) guarantees that any $R_{\text{dep}} \in [0, r_0]$ satisfies the above condition.

B. Intruder strategy

Let $\theta(t_{\text{asym}})$ denote the polar angle between the two agents at the beginning of the asymmetric information phase (see Fig. 4). The intruder wants to move in such a way that it can either breach in this phase, or at least optimize the initial configuration of the *engagement phase* that follows next. In this phase, the position of the intruder is perfectly known to the defender. Therefore, the longer the intruder takes to move towards the target boundary, the higher the chances become for the defender to capture the intruder. Deviating to either side is as likely to be harmful as it is helpful, because assuming the circular probabilistic deployment of Sec. V, there is equal chance of θ and $-\theta$. Further, the intruder has no information on which to base a decision to turn back and try to escape. Therefore, for the intruder's behavior, we make the following conjecture:

Conjecture 2. *The optimal intruder strategy in the asymmetric information phase is to breach in minimum time, i.e., a radial motion towards the center of the target.*

C. Defender strategy

The defender's goal is not to reach the intruder in minimum time. This is because such a strategy may lead to the early termination of the asymmetric information phase, resulting in the configuration that allows the intruder to escape (this scenario, described by Lemma 2, is demonstrated in Sec. VI). Instead, the defender should either simply guard the target (if (7) is true), or lure the intruder sufficiently close to the target so that it can neither breach nor escape (if (10) is true; *capture* configuration in Fig. 3 and Conjecture 1). In either case, the critical configurations are shown in Figure 4. First, it shows the location of each agent at time t_{asym}, when the intruder has just entered the TSR. Second, it shows a hypothetical configuration at time t_{eng} with three key features:

1) Both agents are at the same azimuth ($\theta(t_{\text{eng}}) = 0$).
2) The Apollonius circle is tangent to $\partial\mathcal{R}_T$.
3) The intruder has just sensed the defender (8).

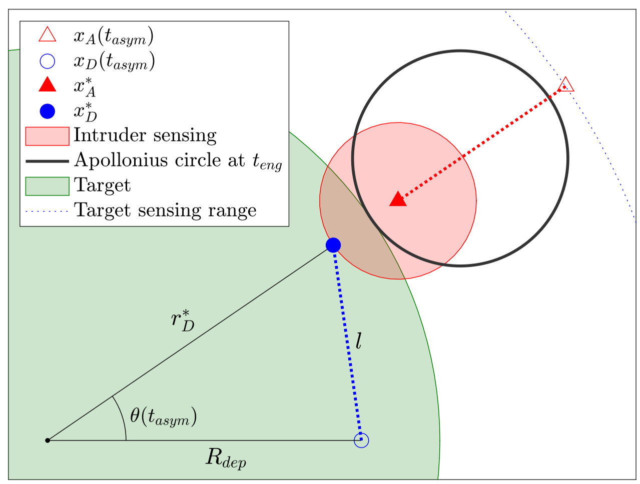

Fig. 4: Critical configuration at time t_{eng}.

In the critical configuration at t_{eng}, we denote the defender position as x_D^*. The radial position of the defender is

$$r_D^* = r_0 - \frac{\rho_A}{1+\nu}. \tag{12}$$

The corresponding intruder radial position at t_{eng} is

$$r_A^* = r_0 + \frac{\nu\rho_A}{1+\nu}. \tag{13}$$

If the defender can reach x_D^* before t_{eng}, then it is sufficient to satisfy Lemma 3. Further, if (10) is true, then the conditions for Conjecture 1 are satisfied. Therefore, reaching this position is sufficient for guard/capture, but showing that it is necessary remains for future work.

Defender strategy, μ_D^{asym}*:* The proposed defender strategy in the asymmetric information phase is to

- Move towards x_D^* at maximum speed;
- If successfully reached x_D^*, stay at x_D^* until the intruder comes within the distance ρ_A.

The performance of this strategy is analyzed next.

D. Performance Guarantees

Depending on the initial conditions of this phase, i.e., the separation angle $\theta(t_{\text{asym}})$ and the deployment radius R_{dep}, the defender may or may not be able to reach the desired position x_D^* in time. Let us define the quantity

$$l_D \triangleq \frac{r_0 + \rho_T - r_A^*}{\nu} = \frac{\rho_T}{\nu} - \frac{\rho_A}{1+\nu}, \tag{14}$$

which denotes the distance that the defender can travel before the attacker reaches the critical distance r_A^*. Note that this is purely in terms of the parameters of the problem.

The next lemma provides the bound on the initial separation angle $\theta(t_{\text{asym}})$ for the defender to guarantee capture.

Lemma 4. *Assume Conjectures 1-2 and that the game parameters satisfy* (10)*. By selecting the deployment radius according to*

$$r_D^* - l_D \le R_{dep} \le r_0 + \rho_T - \rho_A \tag{15}$$

the strategy μ_D^{asym} *guarantees defender's win if* $|\theta(t_{asym})| \le \Theta(R_{dep})$*, where*

$$\Theta(R_{dep}) = \cos^{-1}\left(\frac{1}{2r_D^*}\left[R_{dep} + \frac{{r_D^*}^2 - l_D^2}{R_{dep}}\right]\right). \tag{16}$$

Proof. The lower bound in (15) ensures that the defender can reach x_D^* at least when $\theta(t_{\text{asym}}) = 0$, while the upper bound is from (11). Now, the distance between the defender's initial position and x_D^* is given by

$$l = \sqrt{R_{\text{dep}}^2 + {r_D^*}^2 - 2R_{\text{dep}} r_D^* \cos\theta(t_{\text{asym}})}. \tag{17}$$

Capture is guaranteed if l is less than l_D defined in (14). The sufficient condition $l \le l_D$ reduces to (16). ☐

Remark 1. *When* $R_{dep} = r_D^* - l_D$*, the term inside the inverse cosine becomes 1, corresponding to* $\Theta = 0$*. For any smaller* R_{dep}*, the defender cannot reach* x_D^* *for any value of* $\theta(t_{asym})$*.*

Remark 2. *Noting that* $l \le l_D$ *can also be written as*

$$4R_{dep} r_D^* \cos^2\left(\frac{\theta(t_{asym})}{2}\right) \ge (R_{dep} + r_D^*)^2 - l_D^2 \tag{18}$$

we see that if $l_D \ge R_{dep} + r_D^*$*, then* (18) *is trivially true for any* $\theta(t_{asym})$*, implying that the defender can win from any initial condition. However,* $l_D \ge R_{dep} + r_D^*$ *can possibly happen if we have* $l_D > r_D^*$*, which reduces to* $\rho_T > r_0\nu$*. This is why we assume* (A1) *in order to avoid the degenerate case where the defender can always win.*

We showed how the outcome of the game may be determined by the initial condition of the asymmetric information phase. Next, we consider the strategies in the deployment phase to achieve the favorable initial conditions.

V. Deployment Phase

The defender's goal in the deployment phase is twofold: (i) avoid being sensed by the intruder outside the TSR, and (ii) set its position so that the initial configuration for the asymmetric information phase is optimized. We assume that the defender deploys from $x_D(0) = 0$ to some point on a circle with radius $R_{\text{dep}} < r_0$. It deploys in such a way that

$$\|x_D(t)\| \le R_{\text{dep}}, \quad \forall t < t_{\text{asym}}. \tag{19}$$

If the defender sticks to this policy, there is no intruder strategy that will allow it to sense the defender without entering the TSR. Therefore, the intruder cannot do anything other than to select some angle and to approach the target, i.e., enter the TSR from some "random" angle. Similarly, the defender must select its azimuthal location without any knowledge about the intruder's position. From the above, we conclude that $\theta(t_{asym})$ *at the end of deployment phase is uniformly random in* $[0, 2\pi]$.

Lemma 5. *Assuming that the defender takes the strategy* μ_D^{asym} *in the asymmetric information phase, the optimal deployment radius for the defender is*

$$R_{dep}^* = \sqrt{{r_D^*}^2 - l_D^2}. \tag{20}$$

Moreover, this strategy gives the following critical angle:

$$\Theta(R_{dep}^*) = \cos^{-1}\left(\sqrt{1 - \frac{l_D^2}{{r_D^*}^2}}\right), \tag{21}$$

where r_D^* *and* l_D *are defined in* (14)

Proof. Based on the expression in (16), it is easy to see that the term inside the inverse cosine is minimized when $R_{\text{dep}} = \sqrt{{r_D^*}^2 - l_D^2}$. By substituting this value in (16), we obtain the expression in (21). ☐

Note that as long as (A1) is true, we have $r_D^* > l_D$, which makes the expression (20) valid.

Remark 3. *The optimal value* (20) *satisfies* (15) *(and* (11)*). Therefore, those constraints did not impose any additional restriction on the strategy.*

Corollary 2. *Assuming* (10)*, Conjecture 2, and strategy* μ_D^{asym}*, the probability that the intruder can win the game is bounded from above by*

$$P_A = 1 - \left(\cos^{-1}\sqrt{1 - \frac{l_D^2}{{r_D^*}^2}}\right) \Big/ \pi. \tag{22}$$

This result follows directly from (21) and the fact that $\theta(t_{\text{asym}})$ is uniformly random over $[0, 2\pi]$.

Remark 4. *Consider the boundary of the parameter regime described by* (A1)*. Whenever* (A1) *is satisfied, the probability is* $P_A > 0.5$ *because the term inside* $\cos^{-1}$ *is positive. However, the violation of* (A1) *immediately leads to* $P_A = 0$ *(as discussed in Remark 2. It is interesting to note this* *discrete jump in the intruder-winning probability from* 0.5 *to* 1 *when the parameter changes infinitesimally.*

Corollary 3. *If the defender uses* μ_D^{asym}*, and condition* (7) *is satisfied, then the intruder has a strategy to win.*

Proof. The intruder can appear on a randomly chosen azimuth on the TSR boundary, approach the target in straight line, to start the engagement phase in its winning configuration with a nonzero probability according to Corollary 2. If the intruder is not in its winning configuration, it can always escape and reset the game, by Corollary 1. By continuing this trial, the intruder will eventually win with probability 1, due to Borel-Cantelli Lemma. □

VI. Illustrative Example

We present an illustrative example to demonstrate how the proposed strategy in the asymmetric information phase, μ_D^{asym}, outperforms a naive strategy that would be optimal in a full-state information game. Fig. 5 shows snapshots of a simulation from time $t = t_{\text{asym}}$, where the defender uses μ_D^{asym}. The parameters are $r_0 = 1$, $\nu = 0.85$, $\rho_T = 0.6$, and

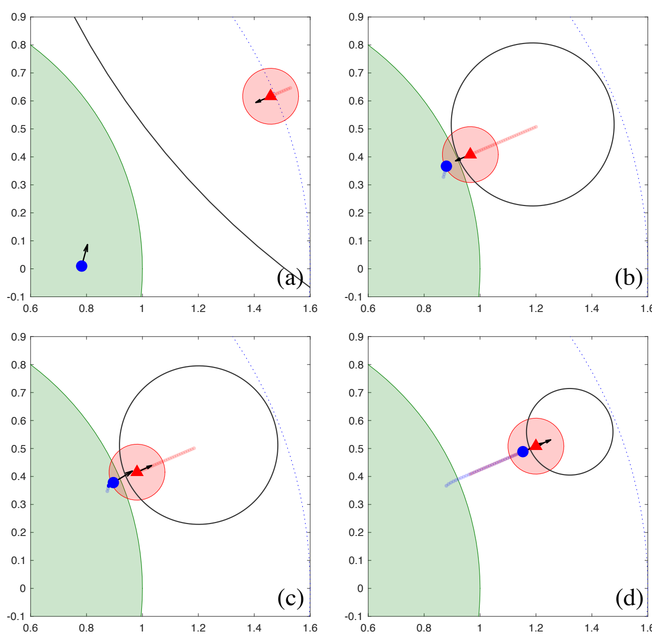

Fig. 5: Proposed strategy. (a) Beginning of the asymmetric information phase. (b) The defender waiting at the critical position at the end of the asymmetric information phase. (c) The intruder senses the defender and game transitions into engagement phase. The Apollonius circle is fully contained in the target sensing region. (d) The engagement phase will end with capture.

$\rho_A = 0.1$. And the initial angular separation is $\theta(t_{\text{asym}}) = 0.4$. Since the Apollonius circle does not intersect with the target at time $t = t_{\text{eng}}$, the intruder cannot win the game. In this simulation, the intruder decides to evade, but eventually get captured (which agrees with Conjecture 1).

In Fig. 6, on the other hand, the defender uses a strategy that would allow it to capture the intruder in minimum time. As shown in the first three snapshots, the defender and the

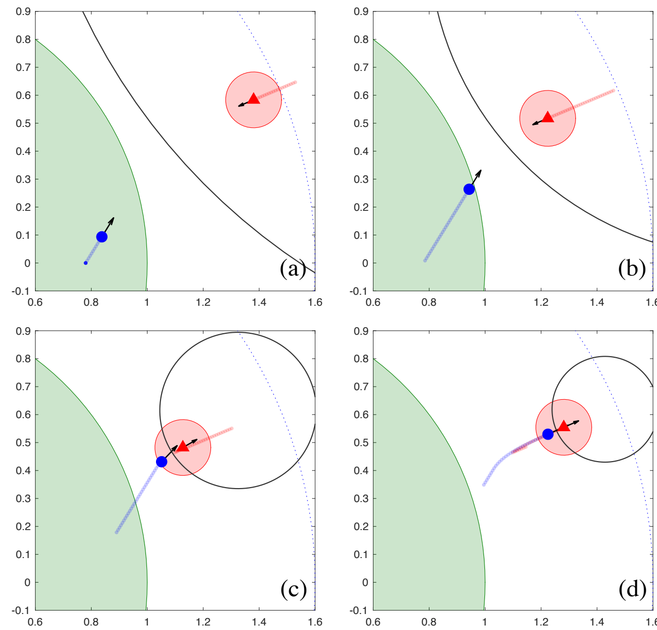

Fig. 6: Naive defender strategy. The defender moves towards the point of fastest interception. However, the engagement phase starts with the configuration guaranteeing evasion.

intruder (who does not see the defender yet) both move towards the collision point in a straight line path. However, since the intruder senses the defender sufficiently close to the boundary of TSR, the Apollonius circle at time $t = t_{\text{eng}}$ sticks outside of the TSR, implying that the intruder can evade. This example demonstrates the importance of taking advantage of the informational asymmetry.

VII. Conclusion

In this paper we formulate a partial information target defense game (PITDG), which is a variant of the target guarding problem that incorporates the agents' sensing constraints. We approached the problem by dividing it into three game phases and considering it in a reverse chronological order. Within the scope of this paper, we focused on a specific parameter regime, along with a particular class of defender strategy, where there is no meaningful searching behavior to achieve informational advantage. The proposed defense strategy places itself in an optimal "waiting" position in the deployment phase so it can reach the desired location during the asymmetric information phase, in order to start the engagement phase from a configuration that guarantees capture. This strategy provides an upper bound on the probability that the intruder can win.

One direction for future extension is to consider the parameter regime where the agents may use searching behavior. This may occur if we relax the assumption (A1) and give more sensing advantage to the intruder, or if the defender has its own local sensing range ρ_D. Additionally, there will be cooperative behaviors if the game is played between teams of agents. We believe that the proposed problem will be useful in exploring various interesting behaviors.

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